

ENHANCING MATHEMATICAL CREATIVITY THROUGH MULTIPLE SOLUTIONS TO OPEN-ENDED PROBLEMS ONLINE¹

Nava L. Livne

Continuing Education, University of Utah
Phone: 801-587-5835, Fax: 801-585-5414
Email address: nlivne@aoce.utah.edu

Oren E. Livne

Continuing Education, University of Utah

Charles A. Wight

Continuing Education, University of Utah

Key Words

Creativity, mathematics, assessment, learning, online, educational technology

Purpose and Objectives

To maintain America's position as a leader in science and technology innovation, higher education institutions must produce a new generation of creative-thinking students. Authorities in mathematics, science, and educational technology, including the National Council of Teachers of Mathematics (2003) and the International Society for Technology in Education (ISTE, 2007) have asserted that tools based on open-ended problems and innovative technologies could stimulate creativity, original thinking and innovation in mathematics and science (Becker & Shimada, 2005; Stansbury, 2007). In contrast, traditional STEM education and assessment focuses on standard curriculum that relies heavily on lower-order thinking skills. Consequently, a high portion of America's High School (HS) students do not have higher-order thinking and creative skills to succeed and excel in college (Alliance for Excellent Education, 2007; Zolli, 2007). Only 32% of HS students qualify to attend four-year College (Education Week, 2008; Greene & Winters, 2005; National Center for Public Policy and Higher Education 2006). They continue to have deficiencies in creative problem-solving in the job market, as indicated by 57% of employers (North American Council for Online Learning and the Partnership for 21st Century Skills, 2006). As the K-12 - Higher Education gap in mathematics continues to widen, achievement and college readiness gaps also persist between white students and those of color, and between middle-class students and those living in poverty: only 9% of all college ready graduates are black, 9% are Hispanic, and 14% are American Indian, compared to 37% of their Asian and White peers (Bozick et. al., 2006; Greene & Forster, 2003).

To help address this need, the University of Utah has developed RUREady (<http://ruready.net>), an individualized learning and self-assessment web site designed to

¹ Copyright © 2008. Livne, Livne, & Wight, */All rights reserved/*. No part of this work may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without written permission from the authors.

improve students' readiness for college-level mathematics courses. This interactive software poses two types of open-ended questions during practice sessions: (1) academic questions that have one standard solution; and (2) creative questions that have at least two different solutions (*e.g.*, brute force vs. elegant, out-of-the-box solution). Utilizing a sophisticated mathematical expression parser (U.S. patent pending), RUPReady provides immediate partial credit scoring and error feedback: each element of the student's response is highlighted as being correct, incorrect, missing, unrecognized or redundant. Using the parser capability, the software evaluates standard and elegant solutions to academic and creative problems. The purpose of the current study is to explore whether *creative thinking in mathematics* can be enhanced through multiple constructed solutions in the RUPReady web environment. Two research questions are posed:

1. Is it difficult for students to find two different solutions to creative problems, distinguished by their originality?
2. Can students' solutions to creative mathematical problems be improved through individualized computer-based practice?

Theoretical Framework

RUPReady is based on previous papers of Livne and associates, indicating that students who approach mathematics with standard or traditional academic types of mathematical thinking skills perform significantly differently from those who approach mathematics with creative thinking in the way they solve mathematical problems (Livne, 2002; Livne & Milgram, 2006). Accordingly, RUPReady was designed to enhance student's mathematical thinking based on four learning components, two of which are relevant to this study (see Livne, Livne, & Wight, 2006 for details):

1. *Types of mathematical thinking*: Based on the 4x4 Structure of Giftedness model (Milgram, 1989, 1991), two abilities reflecting important foundations for talent development are assessed: (a) academic thinking, which refers to standard, analytic ability to consistently reason with mathematical logic; and (b) *creative thinking* that refers to the ability to perceive complex patterns and relationships in original ways, generate ideas that produce multiple original solutions to mathematical problems, and evaluate the solutions' quality (Livne & Milgram, 2006; Munro, 2000; Smith & Stein, 1998). Creative thinking is measured by multiple solution paths and/or solutions, where one solution is standard and another is an original solution (Findell, Gavin, Greenes, & Sheffield, 2000; Glazer, 2001; Livne, Livne, & Milgram, 1999; Munro, 2000; Sheffield, 1999; Williams, 2002). Empirical evidence shows that students' creative mathematical thinking is manifested by multiple original solutions to open-ended mathematical problems, and is indicative of a similar real-world problem solving behavior (Livne, 2002; Livne & Milgram, 2006).
2. *Learning model*: Student's learning is modeled by a learning curve, which postulates a power-law relationship between the learner's mastery of skills and the amount of practice (Liu & Yang, 2005; Answers.com, 2007). The power-law relationship has been supported by many studies and depends on the individual learner's learning rate parameters (Gallistel, Fairhurst, & Balsam, 2004; Liu & Yang, 2005; Ritter & Schooler, 2002). RUPReady uses an *inverse learning curve*

model to estimate students' individual learning rate based on their performance over time, and adaptively presents more problems on their weak topics.

Based on this theoretical framework, a pilot study was conducted to examine whether creative mathematical thinking could be measured by two different solutions, and enhanced by RUReady interactive practice sessions.

Research Methods

Participants

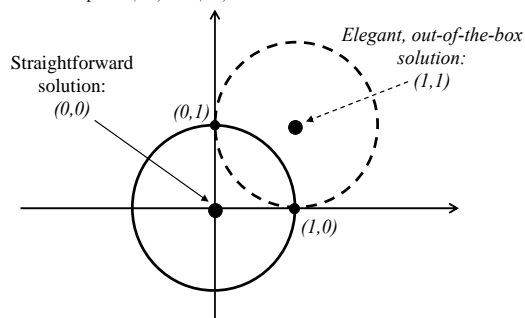
A sample of 112 high school students (47 males and 65 females) 11th and 12th students (mean age = 17.50, $SD = .08$), representing a wide range of abilities, was drawn from three urban public schools in Salt Lake City. Partnerships with these schools were initiated and supported by the Utah State Office of Education Superintendent and the Director of Curriculum Development. As a result, mathematics teachers in these schools committed their students to work through RUReady program.

Tool and Measures

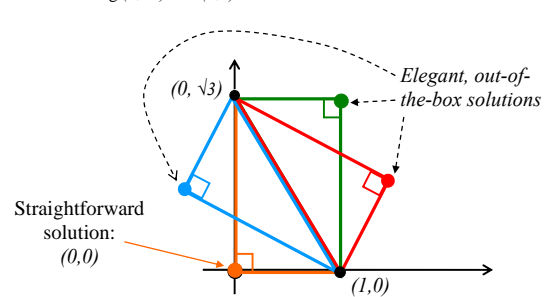
The RUReady online learning and self-assessment program provides opportunities for students to improve their mathematical skills for Intermediate Algebra through solving open-ended mathematical questions in four practice levels. Each session includes a batch of eight open-ended practice questions at the same level of mathematics complexity (1-4). Four questions measure academic thinking and four measure creative thinking. The difference between academic and creative questions is the number of distinct solutions: whereas an academic question has one standard solution, a creative question has one straightforward solution and a second *elegant, out-of-the-box* solution. An example of an academic question is: "Expand the expression $(x-y)^2$ ", which has one standard answer: $x^2 - 2xy + y^2$. An example of a creative question is depicted in Figure 1a: "Find the coordinates of the center of a circle of radius $r = 1$ that also contains the points $(1,0)$ and $(0,1)$ ". The straightforward solution is $(0,0)$, based on the approach taken by most people that centers the circle at the origin; in contrast, $(1,1)$ is a more elegant, out-of-the-box solution that steps outside the natural frame indicated by the axis. A creative question may have one straightforward solution and *multiple* elegant solutions (see Figure 1b).

Figure 1: Two types of solutions to creative mathematical questions.

(a) Find the coordinates of the center of a circle of radius $r = 1$ that also contains the points $(1,0)$ and $(0,1)$.



(b) Find the third vertex of a 30-60-90 triangle whose hypotenuse is the line connecting $(0,\sqrt{3})$ and $(1,0)$.



To ensure *content validity* (Messick, 1995) that refers to the content relevance and quality of the question types, three mathematicians independently reviewed each question in the database and identified its type according to the solutions. The results showed that there was a strong agreement among them on the type of each question, and in particular, on different solutions to creative questions that were distinct (Kendall's coefficients of concordance was $W = .830, p < .0001$; see Seigel & Castellan, 1988), regardless of the questions' difficulty level.

Because the data obtained for this study were limited, only the first and last practice sessions at level 1 were used to assess student performance. Four measures of performance and originality in solving creative questions were defined:

- *IS (initial straightforward score)*: the mean scores of straightforward solutions in the four creative questions of the first practice session.
- *FS (final straightforward score)*: the mean scores of straightforward solutions in the four creative questions of the last practice session.
- *IE (initial elegant score)*: the mean of scores elegant solutions in the four creative questions of the first practice session.
- *FE (final elegant score)*: the mean scores of elegant solutions in the four creative questions of the last practice session.

Scoring. Each of the two solutions to a creative question was automatically scored on a continuous scale of 0-100. The score was the maximum score over all responses; a response score was computed as 100 times the weighted sum of its *overall correctness* (right=0 or wrong=1), and the *fraction of correct elements*, relative to total number of response elements. The scoring was performed automatically in real-time by the aforementioned parser, and was in line with the National Assessment Governing Board (2004)'s recommendation to score mathematical solutions according to their degree of completeness. The scoring model was validated against expert human graders (Livne, Livne, Wight, 2007d), as recommended by the literature (Bennett & Bejar, 1998). Three mathematicians independently scored a sample of 207 representative real-world responses of high school students that worked on the site. The results indicated that there was a strong agreement among the human scorers on scoring each response (Kendall's coefficient of concordance was $W = .890, p < .0001$), regardless of the question type and question's level of difficulty. Stepwise linear regression was used to predict the human score by two RUPready scoring model components, overall correctness and fraction of correct elements. Interestingly, the fraction of correct elements was the better predictor, explaining 77.3% of the variance. The overall correctness component contributed an additional 5.8%. With optimal weighting of the two scoring components, the parser scoring model explained an impressive 82.5% of the human scoring variance, which is equivalent to a human-parser score correlation of $r = .910, p < .0001$! (Livne, Livne, Wight, 2007d). In the current study, students' scores also included a penalty of 6% for each hint they used during problem solving (up to four hints per question).

Procedure

Students created accounts in the RUPready web site and selected the section of the site to prepare for Intermediate Algebra. Next, each student was recommended to take a

diagnostic test to gauge his/her current level of knowledge that resulted in feedback on areas of strengths and weaknesses. Students then engaged in practice sessions. Each session included a batch of eight open-ended practice questions at the same level of mathematics complexity (1-4). Four questions measured academic or analytical thinking and four assessed creative thinking. The student was asked to propose free-form responses to each question in the form of expressions, equations, and numerical answers. Immediate error feedback was provided by the parser following each response: its elements were highlighted in different colors corresponding to each error type, and the total number of correct and erroneous elements of each type was reported. To achieve gradual practice that controls mastery of basic knowledge before moving to more advanced levels (Ausubel, Novak, & Hanesian, 1978), sessions at every difficulty level could be taken unlimited number of times. Questions were selected from a large database and were “parametric” (i.e., used randomly selected input values in their formulation), so that each practice session contained different questions even if a certain difficulty level had been practiced many times. During practice sessions, the student could take repeated the diagnostic tests. For each test performance, the practice questions were adapted to the student’s individual weaknesses, so that the lower a score for a particular topic, the more questions on that topic were presented to the student in subsequent practice sessions. There was no time limit to finish the practice sessions; however, the student’s session timed out after 30 minutes of inactivity.

Creativity was promoted in two ways: when students found a solution to a creative practice question, they were prompted to look for another. In the assessment test, the test instructions stated that half of the questions had two solutions. However, students were not aware *which* questions were creative, thus they were encouraged to look for multiple solutions to *all* questions.

Preliminary Results

Due to the relatively small sample size ($N = 112$), the results should be regarded as preliminary. The two research questions were examined based on the data of Table 1.

Table 1: Means and Standard Deviations (SDs) of four measures of students’ performance in creative questions.

Types of creative solution	Initial	Final
Straightforward solution	$IS = 27.81 \pm 29.90$	$FS = 33.57 \pm 32.71$
Elegant solution	$IE = 12.67 \pm 19.18$	$FE = 16.77 \pm 24.27$

Is It Difficult for Students to Find Two Different Solutions to Creative Problems that Are Distinguished by their Originality?

To examine students’ ability to find two solutions to creative problems, the differences between the mean scores of straightforward and elegant solutions were examined by means of two t-tests for paired samples in the first and last sessions of level 1 practice. The mean scores of *IS* and *IE* were significantly different ($t_{(df=111)} = 5.238, p < .0001$), and so were the mean scores of *FS* and *FE* ($t_{(df=111)} = 5.608, p < .0001$). These findings supported the first research question, indicating that it was more difficult for

students to find elegant solutions to creative problems than straightforward solutions, thereby confirming the classification of the two solution types in RUREady.

Can Students' Solutions to Creative Mathematical Questions be improved through Individualized Computer-Based Practice?

Next, students' *improvement* in finding the two solution types through individualized practice sessions with interactive error feedback was examined. Accordingly, the differences between the mean solution scores in the first and last session (i.e. FS vs. IS and FE vs. IE) were examined using general linear model for repeated measures. The results indicated that these mean scores were significantly different ($F_{(df=1/3)} = 24.400, p < .0001$). Specifically, the mean FS ($M = 33.57$) was significantly higher than the mean IS ($M = 27.81$). Similarly, the mean FE (16.77) was significantly higher than the mean IE ($M = 12.67$). The findings indicated that students improved their scores for *both* straightforward and elegant solutions through interactive practice sessions. However, the large SDs of the four solution scores indicated that there was a large variability in student performance: some of the students solved creative problems very well, finding both straightforward and elegant solutions, whereas others encountered difficulties in finding straightforward solutions, and even more in finding elegant solutions. This might suggest that while students have not been previously trained to tackle creative problem solving in mathematics, they somewhat overcame their difficulties by taking practice sessions.

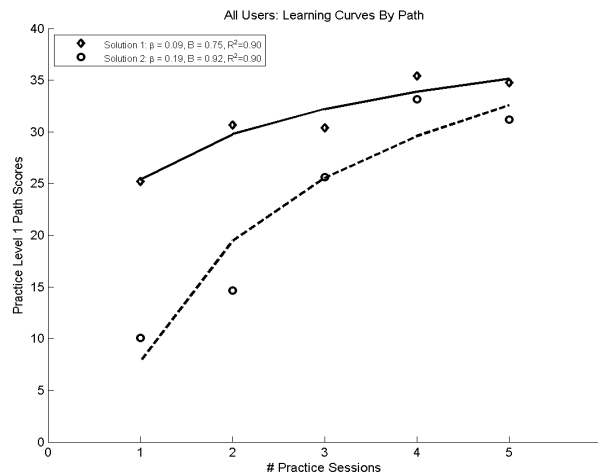
To determine whether there was a difference in the rate of improvement in finding straightforward versus elegant solutions, two Difference mean scores were computed: $D_1 = FS - IS$ and $D_2 = FE - IE$. A t-test for paired samples showed that there was no difference between the mean scores of D_1 and D_2 over the entire student population. This indicated that the practice sessions equally enhanced finding different solutions to creative questions, regardless of their originality. However, it was suspected that these findings could not reflect the actual effectiveness of practice sessions, due to the relatively small number of sessions ($M = 2.48, SD = 2.45$) taken by most participants.

Notwithstanding, for 13 students who worked through five practice sessions or more and were exposed to repeated interactive error feedback, the results were different. Using GLM analysis for repeated measures, the mean score of elegant solutions for the fifth sessions was significantly higher than that for the first session ($F_{(df=1/3)} = 2.930, p < .041, FE = 34.031 \pm 8.787, p < .041$ vs. $IE = 13.938 \pm 5.682$, respectively). In contrast, the corresponding mean scores of straightforward solutions did not differ. Overall, the findings indicated that the higher the number of RUREady practice sessions, the more effective they were for improving the mean scores of elegant solutions compared to those of straightforward solutions. Moreover, the more students practiced, the smaller the SD in their scores became, indicating that practice helped them to find elegant solutions for any of the creative problems.

To further examine the effectiveness of RUREady practice sessions on the learning rates of these 13 students, a learning curve model was fitted for each of the two solution types using logistic regression. The model fit was very high ($R^2=0.90$),

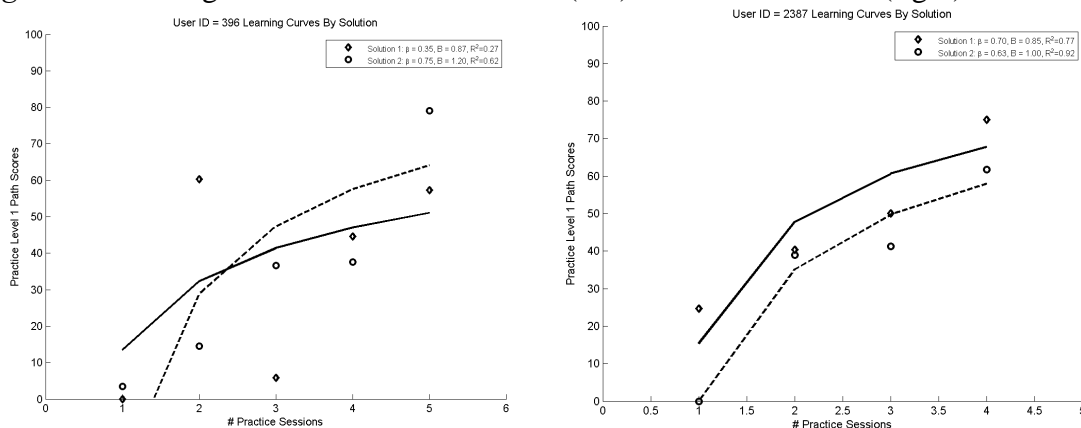
indicating that student learning could be accurately described by a learning curve. Specifically, students improved their performance for both types of the solutions with more practice. Nevertheless, the rate of improvement was twice higher (learning rate coefficient of $\beta = .19$) for elegant solutions than that for straightforward solutions ($\beta = .09$; see Figure 2). Although the initial scores were significantly lower for elegant solutions, the higher learning rate roughly equalized the scores of both solutions after five sessions only.

Figure 2: Learning Curves for Straightforward vs. Elegant Solutions to Creative Mathematical Problems.²



Through the RUPReady individualized computer-based practice, different students exhibited different learning rates for elegant versus straightforward solutions. For example, one creative female learner not only improved her elegant solutions more rapidly than her straightforward solutions, but also attained more elegant solutions with more practice (Figure 3a); in contrast, another male student improved his straightforward and elegant solutions at the same learning rate, however his straightforward solutions always scored higher than his elegant solutions (Figure 3b).

Figure 3: Learning curves of a female learner (left) and a male learner (right).



² Comparison data (diamonds) and learning curve model prediction (solid line) of Solution 1 scores with the corresponding Solution 2 scores (circles and dashed line, respectively).

These preliminary results supported the second research question, indicating that through individualized self-paced practice sessions, different learners can improve their creative mathematical solutions in mathematics at their own pace.

Moreover, all students improved their *scores* by 10% across academic and creative solutions through the practice sessions; their scores with practice were significantly higher than those with no practice ($F_{1/6285} = 7.197, p < .007, M = .61, SD = .4001$, and $M = .56, SD = .4001$, with and with no practice, respectively). Students also increased their *engagement* by 22% with practice, as measured by the number of responses they submitted for each question ($F_{1/6285} = 94.617, p < .0001, M = 2.48, SD = 1.936$ and $M = 2.04, SD = 2.760$, with vs. with no practice, respectively). Interestingly, *minority* students gained more from the practice sessions for *creative* questions than their counterparts. In particular, they increased their *scores* by 11% ($M = 0.55, SD = 0.367$ and $M = 0.49, SD = 3.811$, with vs. with no practice, respectively); in contrast, their white and Asian peers increased the corresponding scores only by 7% ($M = 0.52, SD = 0.366$ and $M = 0.48, SD = 0.370$, with vs. with no practice, respectively). Similarly, *minority* students increased their mathematical *engagement* in solving creative questions by 33% ($M = 3.05, SD = 2.506$ and $M = 2.30, SD = 1.810$, with vs. with no practice, respectively); however, their white and Asian peers increased the corresponding engagement only by 20% ($M = 2.86, SD = 2.506$ and $M = 2.30, SD = 1.810$, with vs. with no practice, respectively). Overall, these preliminary results indicated that RUPReady is an effective tool for improving students' mathematics understanding.

Educational Importance

College readiness concerns both K-12 and higher education institutions. Learner-centered outreach programs such as RUPReady play an increasingly important role in this arena, because they are based on innovative individualized learning and assessment technologies that address the ISTE's National Educational Technology Standards (NETS) for students and teachers in the mathematics and science fields (Watson, 2007). As online learning becomes more a strategic resource for K-12 and higher-education institutions to supplement traditional face-to-face instruction, education leaders discuss how online learning can support minority students' instructional needs: "online learning is not just some remedial course given to minority or disadvantaged students, it's a high-quality education that meets individual needs" (Stansbury, 2008). To this end, RUPReady tools truly engage students in mathematic learning. These tools are timely and provide a groundbreaking solution to help *all* students, especially minority student populations, to develop their creative thinking and problem-solving strategies; thereby bridging individual academic gap between high school and college.

Version 1.0 of the RUPReady software was officially launched in August 2006, and has served to date more than 3,300 students from 43 states and 28 countries, at no cost to the students. This version is limited to scoring only the final answer of each question, as opposed to examining all of intermediate steps that the student may outline. This limitation is particularly evident when creative open-ended problems that have

multiple solution paths but only one final answer could then be employed (Becker & Shimada, 2005), revealing the level and the quality of student mathematical understanding (Magone, Cai, Silver, Wang, 1994; Moon & Schulman, 1995). The next release (RURReady Version 2.0) is currently being developed and scheduled for deployment in the fall of 2008. RURReady 2.0 is a more advanced web application, and has a simpler and more intuitive student interface. The authors plan longitudinal studies to investigate the capability of the RURReady program to predict student success in specific college-level courses in mathematics.

Overall, RURReady provides fundamentally new types of automated learning tools that challenge, stimulate, and guide students via immediate error feedback and unlimited low-stakes assessment opportunities, to accelerate their STEM learning and college readiness. The unrestricted use of the site via the Internet will increase access for underrepresented groups by creating unique new learning frameworks to serve the needs of different types of learners. Given the globalization trend in education, business, and technology, a success of RURReady in increasing the number and diversity of students entering STEM fields will pave the way for a new generation of online “*learnological* systems”. These systems will promote creativity and help eliminate bottlenecks to future advances in science and technology.

References

- Alliance for Excellent Education (2007, September 12). High school teaching for the twenty-first century: Preparing students for college. *Resources for the Future: Issue Brief*. Retrieved, September 25, 2007 from <http://www.all4ed.org/publications/HSTeach21st.pdf>
- Answers.com (2007). *Learning curve*. Retrieved September 21, 2007, from <http://www.answers.com/topic/experience-curve-effects?method=22>
- Ausubel, D. P., Novak, J. D., & Hanesian, H. (1978). *Educational psychology: A cognitive view* (2nd ed.). New York: Holt, Rinehart and Winston.
- Becker, J.P., & Shimada, S. (2005). *The open-ended approach: A new proposal for teaching mathematics*. Reston, VA: National Council of Teachers of Mathematics.
- Bennett, R. E., & Bejar, I. I. (1998). Validity and automated scoring: It's not only the scoring. *Educational Measurement: Issues and Practice*, 17(4), 12-17.
- Bozick, R., Lytle, T., Siegel, P. H., Ingels, S. J., Rogers, J. E., Lauff, E., & Planty, M (2006). *Education Longitudinal Study: 2002/2004* (NCES 2006351). National Center for Education Statistics, Institute of Education Sciences, U.S. Department of Education. Washington, DC.
- Findell, C., Gavin, C., Greenes, C., & Sheffield, L. J. (2000). *Awesome math problems for creative thinking*. (Series of 6 mathematics problem solving books for grades 3 - 8) Chicago, IL: Creative Publications.
- Education Week. (2008, March 27). *Technology counts: STEM: The push to improve science, technology, engineering, and mathematics*. Retrieved April 4, 2008, from <http://www.edweek.org/ew/articles/2008/03/27/30intro.h27.html>
- Gallistel, C. R., Fairhurst, S., & Balsam, P. (2004). The learning curve: Implications of a quantitative analysis. *Proceedings of the National Academy of Sciences*, 101(36), 13124-12131. Retrieved, August 25, 2007, from http://rucss.rutgers.edu/faculty/GnG/The_Learning_Curve.pdf
- Glazer, E. M. (2001b, July 3-6). *Using web-based resources to promote critical thinking*. Paper presented at the 2001 New Zealand Association of Mathematics Teachers (NZAMT) Annual Conference. Wellington, New Zealand. Retrieved February 21, 2008, from <http://math.unipa.it/~grim/AGlazer79-84.PDF>
- Greene, J. P., & Forster, G. (2003). *Public High School Graduation and College Readiness Rates in the United States*. Center for Civic Innovation at the Manhattan Institute. Retrieved April 12, 2008, from

- <http://www.cherrycommission.org/docs/Resources/Preparation/College%20Readiness.pdf>
- Greene, J. P., & Winters, M. A. (2005, February). *Public high school graduation and college-readiness rates: 1991-2002*. (Education Working Paper No. 8). New York, NY: Center for Civic Innovation at the Manhattan Institute. Retrieved April 30, 2008, from http://www.manhattan-institute.org/html/ewp_08.htm
- International Society for Technology in Education (2007). *National educational technology standards for students* (2nd ed.). ISTE, International Society for Technology in Education, Washington, D.C.
- Livne, N. L. (2002). Giftedness in mathematics as a bi-dimensional phenomenon: Theoretical definition and psychometric assessment of levels of academic ability and levels of creative ability in mathematics. (Doctoral dissertation, Tel Aviv University, Israel 2002). *Dissertation Abstracts International*. Retrieved October 28, 2006, from: <http://www.tau.ac.il/education/toar3/archive/etakzir2003-5.htm>
- Livne, N. L., Livne, O. E. & Milgram, R. M. (1999). Assessing academic and creative abilities in mathematics at four levels of understanding. *International Journal of Mathematical Education in Science and Technology*, 30(2), 227-242.
- Livne, N. L. & Milgram, R. M. (2006). Academic versus creative giftedness in mathematics: Two components of the same construct? *Creativity Research Journal*, 18(2), 199-212.
- Livne, N., Livne, O. & Wight, C. (2006). Automated error analysis through parsing mathematical expressions in adaptive online learning. In T. Reeves & S. Yamashita (Eds.), *Proceedings of World Conference on ELearning in Corporate, Government, Healthcare, and Higher Education 2006* (pp. 1321-1325). Chesapeake, VA: AACE.
- Livne, N. L., Livne, O. E. & Wight, C. A. (2007d). Can automated scoring surpass hand grading of students' constructed responses and error patterns in mathematics? *MERLOT Journal of Online Learning and Teaching*, 3(3), pp. 295-306.
- Liu, H-I, & Yang, M-N. (2005). QoL guaranteed adaptation and personalization in E-Learning Systems. *Institute of Electrical and Electronics Engineers (IEEE) Transactions on Education*, 48(4), 676-687.
- Magone, M., Cai, J., Silver, E. & Wang, N. (1994). Validating the cognitive complexity and content quality of a mathematics performance assessment." *International Journal of Educational Research*, 12, 317-340.
- Messick, S. (1995). Validity of psychological assessment: Validation of inferences from person's responses and performance000s as scientific inquiry into score meaning. *American Psychologist*, 50(9), 741-749.

- Milgram, R. M. (1989) (Ed.). *Teaching gifted and talented learners in regular classrooms*. Springfield, IL: Charles C. Thomas. 5
- Milgram, R. M. (1991) (Ed.). *Counseling gifted and talented children: A guide for teachers, counselors, and parents*. Norwood, NJ: Ablex.
- Moon, J. & Schulman, L. (1995). *Finding the connections: Linking assessment, instruction, and curriculum in elementary mathematics*. Portsmouth, NH: Heinemann Publisher.
- Munro, J. (2000). Mathematical giftedness and talent: Thinking creatively in mathematics. *Exceptional Learning and Gifted Education*, 4, 19-24.
- National Assessment Governing Board. (2004). *Mathematical framework for the NAEP assessment 2005*. U.S. Washington, D.C.: U.S. Department of Education National Assessment of Educational Progress.
- National Center for Public Policy and Higher Education (2006). *Measuring up 2006: The national report card on higher education*. Retrieved, April 14, 2008, from http://measuringup.highereducation.org/_docs/2006/NationalReport_2006.pdf
- National Council of Teachers of Mathematics (2003). *The use of technology in the learning and teaching of mathematics*. Reston, VA: National Council of Teachers of Mathematics. Retrieved August 26, 2007, from http://www.nctm.org/uploadedFiles/About_NCTM/Position_Statements/technology.pdf
- North American Council for Online Learning and the Partnership for 21st Century Skills (2006). *Virtual schools and 21st century skills*. Retrieved April 12, 2008, from http://www.nacol.org/docs/NACOL_21CenturySkills.pdf
- Ritter, F. E., & Schooler, L. J. (2002). The learning curve. In *International encyclopedia of the social and behavioral sciences* (pp. 8602-8605). Amsterdam: Pergamon. Retrieved December 12, 2006, from: <http://www.iesbs.com/>
- Sheffield, L. J. (1999). *Developing mathematically promising students*. Reston, VA: National Council of Teachers of Mathematics.
- Seigel, S., & Castellan, N. J. Jr. (1988). *Nonparametric Statistics for the Behavioral Sciences* (2th Edition). New York: McGraw-Hill.
- Smith, M. S., & Stein, M.K. (1998). Selecting and creating mathematical tasks: From research to practice. *Mathematics Teaching in the Middle School*, 3, 344-349.

- Stein, M.K., Smith, M. S., Henningsen, M.A., & Silver, E. S. (2000). *Implementing standards-based mathematics instruction*. New York, NY: Teachers College Press.
- Stansbury, M. (2007). *Teachers gain tech skills while in their PJs Home-based professional development is big success for La. District*. eSchool News Online. Retrieved, August 27, 2007. from <http://www.eschoolnews.com/news/showstoryts.cfm?Articleid=7367>
- Stansbury, M. (2008). *Panelists: Online learning can help minority students. Educators discuss how online courses can help meet underserved students' needs and fulfill course requirements*. Retrieved, April 11, 2008, from <http://www.eschoolnews.com/news/top-news/?i=53470; hbguid=3ec1f1e1-e9e0-40ac-9fcc-52b96065b86b&d=top-news>
- Watson, J. F. (2007, April). *A national primer on K-12 online learning*. North American Council for Online Learning, Evergreen Consulting Associates. Retrieved, April 11, 2008, from: http://www.nacol.org/docs/national_report.pdf
- Williams, G. (2002, July 7-10). Identifying tasks that promote creative thinking in mathematics: A tool. *Proceeding of the 25th Mathematical Education Research Group of Australasia Conference MERGA, 2*, 698-705. Auckland, New Zealand. Retrieved March 21, 2008, from http://extranet.edfac.unimelb.edu.au/DSME/lps/assets/MERGA02_Williams.pdf
- Zolli A. (2007). *Innovation a key theme at NECC 2007*. eSchool News Online. Retrieved, July 19, 2007, from <http://www.eschoolnews.com/news/showstoryts.cfm?Articleid=7232>